Uncertainty Quantification in CFD

Introduction and Motivation

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What is uncertainty quantification (UQ)?

Definition (suggested):

> *The use of both numerical and statistical tools to model problems in physics and engineering.*

- Numerical analysis (and CFD) - knowledge by reasoning:
  - Deterministic PDE describing idealized physical model.
  - Deterministic numerical method providing solution (CFD).
- Statistics - knowledge by data!:
  - Very simple mathematical model - e.g. linear maps.
  - Large amounts of data inform coefficients of the model.

RATIONAL versus EMPIRICAL!

Rational vs Empirical – Example

Example: Sky-diver

- Rational approach:
  - Newton's laws; force balance; ODE.
  - Analytic or numerical solution of ODE.
- Empirical approach:
  - Build setup; perform experiment.
  - Note down results, perhaps variety of materials/blocks.
  - Fit line/curve to data (regression).

Sky-diver

What is uncertainty quantification (UQ)?

UQ combines these complementary approaches to knowledge

$$\Longrightarrow$$

in order to get better understanding (and prediction) of truth.

UQ Approach A: Uncertainty propagation

A. Uncertainty propagation

- Start with rational approach.
- Accept a *lack-of-knowledge* of conditions/physics.
  - Represent *uncertain* values with probability.
  - Deterministic would be: Block mass $m = 10\,\mathrm{kg}$.
  - Stochastic: Coefficient of (kinetic) friction $\mu \sim \mathcal{B}(\alpha, \beta)$ a random-variable.

UQ Approach B: Data-assimilation

B. Data assimilation

- Start with empirical approach.
  - Utilize complex/complete physical simulation, instead of line/curve.
  - Describe stochastic relationship between model and data (*statistical model*).
  - Find parameters matching simulation to data.

Probability as uncertainty

Question:

- What do we mean by probability?

Probability can be used to describe either:

- Inherent, irreducible physical randomness in a system (*aleatoric uncertainty*), OR
- A lack-of-knowledge of a determininistic value (*epistemic uncertainty*).

Epistemic uncertainty

- Depends on the person/observer.
- Can be reduced by learning/measuring more information... or just thinking!
- More natural: can describe probabilities of events that only happen once.

Practicalities

To actually do these:

- Characterize uncertainty in simulation/experiment.
- Propagate pdfs through simulation codes.
- Match simulations to data.

We need statistical models and numerical methods.

> *UQ is the development of complex statistical models for science and engineering, evaluated with numerical methods.*