Part I

Module 1

1.1. Question 1

The correct answer is A: if we have 5 digits, our steps are of the size $0.0001 = 10^{-4}$. This step size is equal to the machine epsilon. If we'd try 0.99999×10^{-4} (answer B), it'd simply get rounded down again. The same applies for answers C and D.

1.2. Question 2

The correct answer is B: if we have the numbers

$$1.0000, 1.0001, ..., 9.9998, 9.9999$$

in the mantissa, then we have

$$\frac{9.9999 - 1.0000}{0.0001} + 1 = 90000$$

distinct numbers in our mantissa. As we have 17 different exponents $(-8, -7, ..., 0, ..., 7, 8)$, and we explicitly add 0 as well, the total number of distinct numbers equals

$$17 \cdot 90000 + 1 = 1530001$$

1.3. Question 3

The correct answer is C: this question may be slightly confusing in the way the program language is written down; first, you should simply read the $\leftarrow$ as =; secondly, you should remember that this for loop simply tells the system to do this specific computation 100 times. In other words, you plug in a certain value for x_0, then the system sets x equal to x_0, then the system computes the square root of this value, then takes the square root again, then takes it again, etc., a hundred times in total. It then squares it a hundred times. So, from a pure mathematical standpoint, you should end up at precisely the same value as you started (because we take the square root a hundred times and then square this a hundred times). However, as we're dealing with a computer system here, it's slightly different.

First, let's see what happens for $x_0 > 1$: let's just try the largest number we can plug in: $x_0 = 9.9999 \cdot 10^8$. If you take the square root a few times with your calculator, you quickly see that we approach 1 at a fast rate. Note that all computations are rounded to 5 digits. Therefore, we'll eventually end up at $\sqrt{1.0001} = 1.00004999$, which will be rounded to 1.0000. This means that if you start squaring it again, you stay stuck at 1, so the final result is 1 as well (it does not "remember" its previous values).

For $0 < x_0 < 1$, let's try the smallest number: $1.0000 \cdot 10^{-8}$: again, take the square root a few times with your calculator and we quickly approach 1. This time, $\sqrt{0.99999} = 0.999995$ (at least, that's what my TI-84+ gives). However, you should now that the more precise value is something like 0.99999498 or so, meaning it is actually rounded down to 0.99999. This means that taking the square root again results in 0.99999, meaning you're stuck in a loop on 0.99999. Then taking square roots will inevitably result in a number that'll be smaller than $1.0000 \cdot 10^{-8}$, meaning it'll be rounded to 0. What if you didn't realize that $\sqrt{0.99999}$ was actually slightly smaller than 0.999995? You simply have to remember that

$$\sqrt{x} < \frac{x+1}{2}$$

for $x < 1$.

1.4. Question 4

The correct answer is B: the Taylor expansion of the cosine is

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

so answer B is correct. Note that even if you did not remember the Taylor expansion of the cosine and you didn't feel like deriving it at the exam, you can already deduce that answers A, C and D must be wrong. The third term in a Taylor series expansion will is already $(x - x_0)^2$, so the third nonzero term must at least be a polynomial of the second degree or higher, meaning A is wrong. Furthermore, there cannot appear a cos (or sin, $\sqrt{x}$, e^x or whatever) in a Taylor series, meaning C and D must be wrong as well.

1.5. Question 5

The correct answer is D: let's just derive the Taylor series for e^x here. Note that $f(1) = f'(1) = f''(1) = f'''(1) = \ldots = e^1 = e$; thus, we have

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n = \frac{e}{0!}(x-1)^0 + \frac{e}{1!}(x-1)^1 + \frac{e}{2!}(x-1)^2 + \frac{e}{3!}(x-1)^3 + \ldots$$

Thus, if we include only the first three terms in the expansion, then the first term in the truncation error is simply $\frac{e}{6}(x-1)^3$, so answer D is correct.

1.6. Question 6

The correct answer is B. Again, you can derive the entire Taylor series but it is not really necessary as we know that all terms in the Taylor series are nonzero and therefore, if we have a 3-term Taylor expansion, $R(x)$ is associated with the fourth term, which will be

$$R(x) = \frac{f^{(3)}(\xi)}{3!} h^3$$

We have that $f^{(3)}(\xi)$ equals

$$f'(\xi) = 2e^{2\xi}$$
$$f''(\xi) = 4e^{2\xi}$$
$$f'''(\xi) = 8e^{2\xi}$$

and thus

$$R(x) = \frac{8e^{2\xi}}{3 \cdot 2 \cdot 1} h^3 = \frac{4}{3} \exp(2\xi) h^3$$

so answer B is correct.

1.7. Question 7

The correct answer is A: the Taylor series expansion for a sine is

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

Thus, the first non-zero term in the truncation error is

$$\frac{x^5}{5!} \quad \rightarrow \quad \frac{\left(\frac{\pi}{2}\right)^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 0.07969$$

1.8. Question 8

The correct answer is A: the Taylor series for e^{-x^2} around $x = 1$ is not immediately obvious probably, so let's just derive it here. We have

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

where we're interested only in the first three terms:

$$\begin{aligned} f(x) &= e^{-x^2} \quad \rightarrow \quad f(1) = e^{-1^2} = e^{-1} \\ f'(x) &= -2xe^{-x^2} \quad \rightarrow \quad f'(1) = -2\cdot 1\cdot e^{-1^2} = -2e^{-1} \\ f''(x) &= -2e^{-x^2} + 4x^2 e^{-x^2} \quad \rightarrow \quad f''(1) = -2e^{-1^2} + 4\cdot 1^2\cdot e^{-1^2} = 2e^{-1} \end{aligned}$$

Thus, the Taylor series is

$$f(0) = \frac{e^{-1}}{0!}(0-1)^0 - \frac{2e^{-1}}{1!}(0-1)^1 + \frac{2e^{-1}}{2!}(0-1)^2$$

so that the magnitude of the first term in the truncation error is

$$\frac{2e^{-1}}{2}(-1)^2 = e^{-1}$$

and thus answer A is correct.

1.9. Question 9

The correct answer is C: let's just do it three times:

$$\begin{aligned} x_0 &= \frac{-2+1}{2} = -0.5 \\ f(0.5) &= e^{0.5} - 1 = 0.6487 \\ x_1 &= \frac{-2+0.5}{2} = -0.75 \\ f(-0.75) &= e^{-0.75} - 1 = -0.5276 \\ x_2 &= \frac{0.5-0.75}{2} = -0.125 \end{aligned}$$

Note that three steps mean three times taking the center of the remaining interval.

1.10. Question 10

The correct answer is C: repeated bisection will *always* converge to one of the roots. Converging to more than one root just doesn't make sense semantically. Answers E and F are also just wrong.

1.11. Question 11

The correct answer is E: we have two (obvious) options; let's just see the results of both of them:

$$\begin{aligned} 5x^2 &= e^x \\ x = \phi(x) &= \sqrt{\frac{e^x}{5}} \\ x_1 = \phi(10) &= \sqrt{\frac{e^{10}}{5}} = 66.37 \\ x_2 = \phi(10) &= \sqrt{\frac{e^{66.37}}{5}} = 1.156 \cdot 10^{14} \end{aligned}$$

which obviously diverges, so let's try the other:

$$\begin{aligned} e^x &= 5x^2 \\ x = \phi(x) &= \ln\left(5x^2\right) \\ x_1 = \phi(10) &= \ln\left(5 \cdot 10^2\right) = 6.2146 \\ x_2 = \phi(6.2146) &= \ln\left(5 \cdot 6.2146^2\right) = 5.263 \end{aligned}$$

so answer E is correct.

1.12. Question 12

The correct answer is A: just basic knowledge, really.

1.13. Question 13

The correct answer is E: just remember the formula

$$\begin{aligned} \phi(x) &= x - \frac{f(x)}{f'(x)} \\ f(x) &= 8^x - 8x^3 \\ f'(x) &= \ln(8) \cdot 8^x - 24x^2 \\ \phi(x) &= x - \frac{8^x - 8x^3}{\ln(8) \cdot 8^x - 24x^2} \\ x_1 = \phi(0.5) &= 0.5 - \frac{8^{0.5} - 8 \cdot 0.5^3}{\ln(8) \cdot 8^{0.5} - 24 \cdot 0.5^2} = 15.936 \\ |x_1 - \tilde{x}| = |15.936 - 1| = 14.936 \end{aligned}$$

So answer E is correct.

1.14. Question 14

The correct answer is B: in general the only way to answer these kind of questions is to perform the iteration (no such question will appear on the Quiz). Letting your calculator do the iteration a few times; the fastest way to do this is by simple use of ans (at least that's how it's called on my TI-30XB):

$$\text{ans} - \frac{8^{\text{ans}} - 8 \cdot \text{ans}^3}{\ln(8) \cdot 8^{\text{ans}} - 24 \cdot \text{ans}^2}$$

If you do this like 10 times (which is just 10 times pressing enter so it's not that much work at all), you quickly see that it converges to $x = 2$.

1.15. Question 15

The correct answer is D: there is little chance you were able to solve this all by yourself probably, though (at least I wasn't). Remember that Newton's method was

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

but how does that work for a system of equations? Then x_1, x_0 and $f(x_0)$ all become vectors:

$$\begin{aligned}
\mathbf{x}_1 &= \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \\
\mathbf{x}_0 &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \\
\mathbf{F} &= \begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix} = \begin{bmatrix} x^2+y^2-1 \\ xy-\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 2^2+(-1)^2-1 \\ 2\cdot -1-\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{9}{4} \end{bmatrix}
\end{aligned}$$

Note that for $\mathbf{F}$, I set the equations equal to zero (as required). Now, what happens with $f'(x_0)$? That becomes the Jacobian matrix:

$$\mathbf{F}' = \begin{bmatrix} f_x(x,y) & f_y(x,y) \\ g_x(x,y) & g_y(x,y) \end{bmatrix} = \begin{bmatrix} 2x & 2y \\ y & x \end{bmatrix} = \begin{bmatrix} 2\cdot 2 & 2\cdot -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 2 \end{bmatrix}$$

where f_x is the partial derivative of f w.r.t. x, etc. Newton's method then becomes

$$\mathbf{x}_1 = \mathbf{x}_0 - \mathbf{F}'^{-1}\mathbf{F}$$

which may be rewritten to

$$\boldsymbol{\Delta}\mathbf{x}_0 = -\mathbf{F}'^{-1}\mathbf{F}$$

Now, computing inverses is generally not something nice, so we rather write this as

$$\mathbf{F}'\Delta\mathbf{x}_0 = -\mathbf{F}$$

Writing all of this out leads to:

$$\begin{aligned}
\begin{bmatrix} 4 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} \Delta x_0 \\ \Delta y_0 \end{bmatrix} &= -\begin{bmatrix} 4 \\ -\frac{4}{9} \end{bmatrix} \\
4\Delta x_0 - 2\Delta y_0 &= -4 \\
-1\Delta x_0 + 2\Delta y &= 2.25
\end{aligned}$$

and so answer D is correct. Please note: you could have solved directly for $\boldsymbol{\Delta}\mathbf{x}_0$ by computing $\mathbf{F}'^{-1}$, but that would lead to explicit solutions for Δx_0 and Δy_0: you'd need to plug in these values in each of the answers to see which set of equations corresponds to this set of solutions.

1.16. Question 16

The correct answer is E. First of all, in the previous question, we found

$$\mathbf{F}' = \begin{bmatrix} 2x & 2y \\ y & x \end{bmatrix}$$

and $\Delta\mathbf{x}_0 = -\mathbf{F}'^{-1}\mathbf{F}$. The determinant equals

$$\det\mathbf{F}' = 2x^2 - 2y^2 = 2\left(x^2 - y^2\right)$$

Now, if $x = y$ (or $x = -y$ for that matter), then $\mathbf{F}'$ becomes singular as the determinant equals zero, so then everything breaks and it diverges.

Part II

Module 2

1.17. Question 1

The correct answer is B. This is the uniqueness of polynomial interpolation, shown by e.g. by $\det V \neq 0$ where V is the Vandermonde matrix.

1.18. Question 2

The correct answer is F. This is the Cauchy theorem for polynomial interpolation error, combined with nodal-polynomial from the Chebychev nodes.

1.19. Question 3

The correct answer is D: we have $n-1+1=n$ conditions to satisfy, so we must use a polynomial of degree $n-1$ to have enough coefficients to satisfy these (if you don't fully understand what I'm saying, suppose we have $i=0,1,2,3$. Then we must use $p(x)=ax^3+bx^2+cx+d$ to make a system with four equations and four unknowns). Thus, the correct answer is D.

1.20. Question 4

The correct answer is C: you may be inclined, based on the previous question, to answer D (that a polynomial of degree 3 is necessary), but question 3 is the minimum number that *guarantees* you to be able to interpolate all these points. Looking at this dataset, we actually see that $p(x)=x^2$ perfectly matches the data: thus, we only need a polynomial of degree 2, and C is the correct answer.

1.21. Question 5

The correct answer is D: for interpolation, you can use *any* function for the base. However, polynomials are simply much easier and are therefore mostly used.

1.22. Question 6

The correct answer is F: remember that we must solve

$$\begin{aligned}
\begin{bmatrix} 1 & 0 & 0 \\ 1 & (x_1-x_0) & 0 \\ 1 & (x-2-x_0) & (x_2-x_0)(x_2-x_1) \end{bmatrix}\begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} \\
\begin{bmatrix} 1 & 0 & 0 \\ 1 & (1-0) & 0 \\ 1 & (2-0) & (2-0)(2-1) \end{bmatrix}\begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} \\
\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}
\end{aligned}$$

From the first row, we see $d_0=4$. Then from the second row, the equation $1\cdot 4+1\cdot d_1=3$ and thus $d_1=-1$. Then, from the third row, we have

$$1\cdot 4+2\cdot -1+2\cdot d_2=1$$

and thus $d_2=-1/2$ and thus answer F is correct.

1.23. Question 7

The correct answer is A: from the text, we have that $n = 1$ (our last node is x_1 after all). For $f(x) = \cos(\pi x)$, we have

$$f''(\xi) = -\pi^2 \cos(\pi\xi)$$

Furthermore, for $x = 1/4$, we have

$$\prod_{j=0}^{1}\left(x - x_j\right) = (x - x_0)(x - x_1) = \left(\frac{1}{4} - 0\right)\left(\frac{1}{4} - \frac{1}{2}\right) = -0.0625$$

Thus, the interpolation error is given by

$$R\left(\cos\pi x; \frac{1}{4}\right) = \frac{f''(\xi)}{(1+1)!} \cdot -0.0625 = -0.03125 \cdot -\pi^2 \cos(\pi\xi) = 0.3084 \cos(\pi\xi)$$

The maximum value for this is attained at $\xi = 0$, for which the interpolation error is 0.31, thus answer A is correct (note: the only restriction on ξ is that it must be between 0 and 0.5).

1.24. Question 8

The correct answer is H: the point of Chebychev grids is to concentrate your datapoints near the edges of the domain. Furthermore, Lagrange and monomial basis result in exactly the same polynomial, thus it wouldn't make any sense at all that they would require different choices of grid.

1.25. Question 9

The correct answer is E: we want a cubic polynomial, so we need four nodes, and thus four zeros, and thus we need a fourth order Chebychev, i.e. $T_4(x)$ and thus $m = 4$. For x_0, we must then use the fourth root as well, so we have

$$\xi_4 = \cos\left(\frac{2 \cdot 4 - 1}{2 \cdot 4}\pi\right) = -0.92388$$

We then have

$$x_0 = \frac{b+a}{2} + \frac{b-a}{2}\xi_4 = \frac{5+1}{2} + \frac{5-1}{2} \cdot -0.92388 = 1.152$$

and thus E is the correct answer.

1.26. Question 10

The correct answer is A: the error does not converge to zero if you keep on increasing the number of data points. That was basically the point of this section.

1.27. Question 11

The correct answer is D: 2 is obviously wrong, 4 is obviously correct. 1 is wrong: whatever basis you choice, you end up at exactly the same polynomial. 3 is correct: choosing x_i closer to the endpoints of the interval will result in a smaller approximation error (that was the entire point this section was making).

1.28. Question 12

The correct answer is A: note that this time you do not have to use that fancy formula, as that merely provides an estimate for the upper bound on the interpolation error on the interval, not the exact error for a certain value of x. Interpolating with a first-order polynomial is easy: we have $f_0(0) = \cos(\pi \cdot 0) = 1$ and $f_1(1/2) = \cos(\pi \cdot 1/2) = 0$. Thus, the interpolation function is simply $p_1(x) = 1 - 2x$ (because this formula goes through those two points). We have

$$\begin{aligned} f\left(\frac{1}{4}\right) &= \cos\left(\frac{\pi}{4}\right) = 0.7071 \\ p_1\left(\frac{1}{4}\right) &= 1 - 2 \cdot \frac{1}{4} = 0.5 \\ \epsilon\left(\frac{1}{4}\right) = \left|f\left(\frac{1}{4}\right) - p_1\left(\frac{1}{4}\right)\right| &= |0.7071 - 0.5| = 0.2071 \end{aligned}$$

and thus answer A is correct.

1.29. Question 13

The correct answer is F: as we are supposed to use a linear polynomial, we are only allowed to use two nodes. However, five data points are given, so which to use? Well, most logically, we'll just use $t = 15$ and $t = 18$. Then, from interpolation, we find

$$v(16) = 24 + \frac{37 - 24}{18 - 15} \cdot (16 - 15) = 28.33$$

and thus answer F is correct.

1.30. Question 14

The correct answer is C: we have $p(x) = d_0 + d_1(x - x_0) + d_2(x - x_0)(x - x_1) = d_0 + d_1 x + d_2 x(x - 1)$. Furthermore, remember that we must solve

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 0 \\ 1 & (x_1 - x_0) & 0 \\ 1 & (x - 2 - x_0) & (x_2 - x_0)(x_2 - x_1) \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & (1 - 0) & 0 \\ 1 & (3 - 0) & (3 - 0)(3 - 1) \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} 10 \\ 11 \\ 14 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} &= \begin{bmatrix} 10 \\ 11 \\ 14 \end{bmatrix} \end{aligned}$$

From the first row, we then easily see that $d_0 = 10$. From the second row, we then straightforwardly have $d_1 = 1$. From the final row, we must then solve

$$1 \cdot 10 + 3 \cdot 1 + 6d_2 = 14$$

and thus $d_2 = 0.1667$. Now we have

$$\begin{aligned} p_2(x) &= 10 + x + 0.1667x(x - 1) \\ p_2(2) &= 10 + 2 + 0.1667 \cdot 2 \cdot (2 - 1) = 12.3333 \end{aligned}$$

and thus answer C is correct.

1.31. Question 15

The correct answer is A: the easy way to find the answer is by simply checking all of the formulas whether they lead to the correct data points; doing so will lead to A being the only correct polynomial. Alternatively, you can just do it the hard way (there are four nodes, thus you need a third order polynomial):

$$p_3 = f_0 \prod_{j=0,j\neq 0}^{3} \frac{x-x_j}{x_0-x_j} + f_1 \prod_{j=0,j\neq 1}^{3} \frac{x-x_j}{x_1-x_j} + f_2 \prod_{j=0,j\neq 0}^{2} \frac{x-x_j}{x_2-x_j} + f_3 \prod_{j=0,j\neq 3}^{3} \frac{x-x_j}{x_3-x_j}$$

where $f_0 = -6$, $f_1 = 0$, $f_2 = -2$ and $f_3 = -12$. Working out the products (as $f_1 = 0$, we don't need to deal with the second product):

$$\begin{aligned}
\prod_{j=0,j\neq 0}^{n} \frac{x-x_j}{x_0-x_j} &= \frac{x-x_1}{x_0-x_1}\frac{x-x_2}{x_0-x_2}\frac{x-x_3}{x_0-x_3} = \frac{(x-1)(x-3)(x-5)}{(-1-1)(-1-3)(-1-5)} \\
&= \frac{x^3-9x^2+23x-15}{-48} \\
\prod_{j=0,j\neq 2}^{n} \frac{x-x_j}{x_2-x_j} &= \frac{x-x_0}{x_2-x_0}\frac{x-x_1}{x_2-x_1}\frac{x-x_3}{x_2-x_3} = \frac{(x--1)(x-1)(x-5)}{(3--1)(3-1)(3-5)} \\
&= \frac{x^3-5x^2-x+5}{-16} \\
\prod_{j=0,j\neq 3}^{n} \frac{x-x_j}{x_3-x_j} &= \frac{x-x_0}{x_3-x_0}\frac{x-x_1}{x_3-x_1}\frac{x-x_2}{x_3-x_2} = \frac{(x--1)(x-1)(x-3)}{(5--1)(5-1)(5-3)} \\
&= \frac{x^3-3x^2-x+3}{48}
\end{aligned}$$

and thus

$$\begin{aligned}
p_3(x) &= -6\cdot\frac{x^3-9x^2+23x-15}{-48} + -2\cdot\frac{x^3-5x^2-x-5}{-16} + -12\cdot\frac{x^3-3x^2-x+3}{48} \\
&= \frac{x^3-9x^2+23x-15}{8} + \frac{x^3-5x^2-x+5}{8} - \frac{x^3-3x^2-x+3}{4} \\
&= \frac{2x^3-14x^2+22x-10}{8} - \frac{2x^3-6x^2-2x+6}{8} = \frac{-8x^2+24x-16}{8} = -x^2+3x-2
\end{aligned}$$

Yet another way would be to just see that the equation is quadratic, as the step sizes decreases linearly (6, -2, -10) (thus the derivative must be linear), and then realizing that it must have a line of symmetry at $x = 1.5$, and then making the appropriate function for it (but this is still harder than just trial and error).

1.32. Question 16

The correct answer is A: we have to solve the matrix equation

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & (x_1 - x_0) & 0 \\ 1 & (x-2-x_0) & (x_2-x_0)(x_2-x_1) \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & (18-15) & 0 \\ 1 & (22-15) & (22-15)(22-18) \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 37 \\ 25 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 7 & 28 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 37 \\ 25 \end{bmatrix}$$

From the first row, we obviously have $d_0 = 24$. From the second one, we then have $d_1 = 13/3 = 4.3333$. We then have for the third row that

$$1 \cdot 24 + 7 \cdot 4.3333 + 28d_2 = 25$$

and thus $d_2 = -1.0476$ and thus answer A is correct.