Applied Numerical Analysis – Quiz #3

Modules 5 and 6

Name: ______________________________ Student number: ______________

DO NOT OPEN UNTIL ASKED

Instructions:

- Make sure you have a machine-readable answer form.
- Write your name and student number in the spaces above, **and** on the answer form.
- Fill in the answer form **neatly** to avoid risk of incorrect marking.
- Fill in the version number of your quiz (see bottom right, A-D) on the answer form.
- Use only pencil on the answer form, and correct with a rubber.
- This quiz requires a calculator.
- Each question has exactly one correct answer.
- Correct answers will receive 5 points, unanswered questions 1 point, incorrect answers 0 points.
- This quiz has **10 questions** and **5 pages** in total.

Question 1 Consider the scalar initial-value problem:

$$u' = f(u), \qquad u(0) = 1$$

We use the following three methods, with the same time step $\Delta t = 1$:

(i) Forward-Euler

(ii) Backward-Euler

(iii) Method of Heun (also-known-as the trapezoidal rule)

In each case we approximate u at $t = 5$. When the method is implicit, the system is solved at each timestep with 10 iterations of a fixed-point iteration (and a initial guess $u_{i+1}^{(0)} = u_i$ – to obtain $u_{i+1}^{(10)}$). What is the total number of evaluations of f needed for each method?

A: Unanswered
B: (i) 5, (ii) 50, (iii) 55
C: (i) 5, (ii) 5, (iii) 5
D: (i) 5, (ii) 50, (iii) 50
E: (i) 5, (ii) 5, (iii) 55
F: (i) 50, (ii) 50, (iii) 5
G: (i) 50, (ii) 50, (iii) 50
H: (i) 50, (ii) 50, (iii) 55

Question 2 Consider the following scheme:

$$y_{n+1} = y_n + \Delta t[\alpha f(y_n) + (1-\alpha)f(y_{n+1})], \quad 0 \le \alpha \le 1$$

which can be regarded as a generalized Heun scheme (for $\alpha = \frac{1}{2}$ it is exactly Heun). Note that, for all α, the stability boundary of this scheme is a circle in the z-plane (for the standard definition of z), with center on the real-line. For what range of α is the method stable for $z = -10$?

A: Unanswered
B: $\alpha < \frac{1}{5}$
C: $\alpha < \frac{2}{5}$
D: $\alpha < \frac{3}{5}$
E: $\alpha < \frac{1}{2}$
F: $\alpha = 0$ or $\alpha = \frac{1}{2}$
G: No such value of α

Question 3 Consider the equation of the harmonic damped oscillator with an external time dependent force:

$$x'' + x' + x = \sin(\pi t)$$

with the initial-conditions $x(0) = 0$ and $x'(0) = 1$. Applying forward-Euler method with a timestep $\Delta t = \frac{1}{2}$, what is the position x_1 and the velocity x'_1 after one iteration?

A: Unanswered
B: $x_1 = x'_1 = 1.0$
C: $x_1 = x'_1 = 0.5$
D: $x_1 = x'_1 = 0.0$
E: $x_1 = x'_1 = -0.5$
F: $x_1 = x'_1 = -1.0$

Question 4 Consider the ODE system

$$x' = 2x + y$$
$$y' = 0x - 3y$$

with initial condition $x(0) = y(0) = 1$. The solution is approximated with forward-Euler with a timestep $\Delta t = \frac{1}{40}$. What can be said about the stability of the ODE and forward-Euler (for this choice of Δt)?

A: Unanswered.
B: No solution of the ODE exists.
C: The ODE is itself unstable.
D: The ODE is unstable but forward-Euler is stable.
E: The ODE is stable and forward-Euler is stable.
F: The ODE is stable and forward-Euler is unstable.

Question 5 Consider the following function:

$$f(x, y) = 3x^2 + y^2 - 3x + 3xy$$

Newton's method for optimization is applied and the following two points are found: $a = (3, 2)$ and $b = (2, -3)$. What can be said about these points?

A: Unanswered
B: a is a maximum and b is a minimum
C: a is a minimum and b is a maximum
D: a is a maximum and b is neither a maximum or minimum
E: a is a minimum and b is neither a maximum or minimum
F: a is neither a maximum or minimum and b is a maximum
G: a is neither a maximum or minimum and b is a minimum

Question 6 Consider the following function:

$$f(x_1, x_2) = x_1^2 + 1000x_2^2$$

that presents a minimum at $x = (0, 0)$. Starting from *any* $x_0 \neq (0, 0)$, M is the number of steepest-descent iterations and N is the number of Newton iterations needed to find the optimum exactly. Which one of the following statements is true?

A: Unanswered
B: $M = 1$, $N = 1$
C: $M \geq 1$, $N = 1$
D: $M \geq 2$, $N = 1$
E: $M \geq 1$, $N = 2$
F: $M \geq 1$, $N \geq 1$
G: $M \geq 2$, $N \geq 1$

Question 7 In an optimization procedure, the steepest descent method is being used. In order to verify that the code has been properly implemented, we will calculate the first stepsize α by hand, to cross check it with the one used by the code. The test function

$$f(x,y) = x^2 + xy + y^2$$

will be used, with the initial point $x_0 = 1$, $y_0 = 1$. What is the value of the first stepsize α taken by the algorithm?

A: Unanswered
B: $\alpha = \frac{1}{3}$
C: $\alpha = -\frac{1}{3}$
D: $\alpha = \frac{2}{3}$
E: $\alpha = -\frac{2}{3}$
F: $\alpha = 1$
G: $\alpha = -1$
H: $\alpha = 2$

Question 8 After an experiment on an airfoil you have a large set of N samples (x_i, y_i) relating the lift y to the angle of attack x. You want to fit this data with a curve of the form: $y = ax + b$. Rather than simple regression to find a and b, you use a fancy statistical approach (called "Maximum-Likelihood Estimation"), which requires maximizing $L(a, b)$:

$$L(a,b) = \sum_{i=1}^{N} \left[-\frac{N}{2}\log(2\pi) - \frac{N}{2}\log(b^2) - \frac{1}{2b^2}(y_i - ax_i)^2 \right],$$

where all logs are natural logs. What is the vector pointing in the steepest *ascent* direction?

A: Unanswered
B: $\left(\sum_{i=1}^{N} \frac{x_i}{b^2}(y_i - ax_i), \sum_{i=1}^{N}(-\frac{N}{b} + \frac{(y_i-ax_i)^2}{b^3})\right)$
C: $\left(\sum_{i=1}^{N} \frac{a^2}{b}(y_i + ax_i), \sum_{i=1}^{N}(\frac{a\cdot N}{b^2} - \frac{(y_i-ax_i)^2}{a^2})\right)$
D: $\left(\sum_{i=1}^{N} \frac{a}{b}(y_i - ax_i)^2, \sum_{i=1}^{N}(\frac{a}{b} + \frac{(y_i-ax_i)}{b^2})\right)$
E: $\left(\sum_{i=1}^{N} \frac{a}{b}(y_i + ax_i)^2, \sum_{i=1}^{N}(\frac{N}{ab} + \frac{(y_i-ax_i)}{b^3})\right)$

Question 9 When using Newton's method to find a minimum of $f(\mathbf{x})$, which of the following are true?

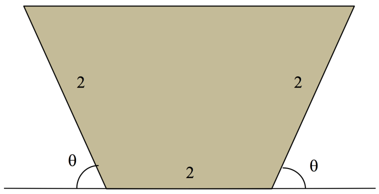
(i) $f(\mathbf{x})$ must be once continuously differentiable at the iterates $\mathbf{x}_i$, but not twice.

(ii) If Newton converges, it is always to a minimum.

(iii) Newton converges to a the same local optimum, independent of the initial guess.

A: Unanswered
B: (i)
C: (ii)
D: (iii)
E: (i),(ii)
F: (i),(iii)
G: (iii)
H: None

Question 10 Consider the gutter-shape plotted below with base- and edges of length 2. Goal is to maximize the cross-sectional area of the gutter by varying the angle θ. Golden-section search is applied to solve this problem using an initial interval $I_0 = [0, \frac{\pi}{2}]$. Apply one iteration of this method to obtain I_1. What is the midpoint of I_1? [Note: In G-S search on the interval $[0, 1]$, the interior sample points are at $\frac{1}{\phi}$ and $1 - \frac{1}{\phi}$ with $\phi = \frac{1+\sqrt{5}}{2}$.]



A: Unanswered.
B: $\frac{\pi}{2}$
C: $\frac{7\pi}{9}$
D: $\frac{\pi^2}{2}$
E: 1.085
F: 1.125