Applied Numerical Analysis – Quiz #3
Modules 5 and 6

Name: ____________________________ Student number: _____

DO NOT OPEN UNTIL ASKED

Instructions:

• Make sure you have a machine-readable answer form.

• Write your name and student number in the spaces above, and on the answer form.

• Fill in the answer form neatly to avoid risk of incorrect marking.

• Fill in the version number of your quiz (see bottom right, A-D) on the answer form.

• Use only pencil on the answer form, and correct with a rubber.

• This quiz requires a calculator.

• Each question has exactly one correct answer.

• Correct answers will receive 5 points, unanswered questions 1 point, incorrect answers 0 points.

• This quiz has 10 questions and 4 pages in total.
Module 5: Numerical solution of ODEs

Throughout we assume the standard form of the ODE: \( u'(t) = f(u(t)) \).

**Question 1**  The ODE \( u' = -\frac{5}{2}u \) with initial condition \( u(0) = 2 \) is solved numerically using (a) the forward-Euler method (explicit), and (b) the backward-Euler method (implicit). In both cases the stepsize \( \Delta t = 0.6 \). Of the two methods, which are stable?

A: Unanswered  
B: Neither  
C: (a) only  
D: (b) only  
E: Both

**Question 2**  A function \( f(u) \) is Lipschitz continuous on an interval \([a, b]\) if: there exists some Lipschitz constant \( 0 < L < \infty \), such that for all \( u_1 \) and \( u_2 \) in \([a, b]\)

\[ |f(u_1) - f(u_2)| \leq L|u_1 - u_2|. \]

The function \( f(u) = u^2 \) is Lipschitz continuous on \([0, 10]\). What is the minimum constant \( L \) needed in the definition above, for this function on this interval?

A: Unanswered  
B: \( \frac{1}{5} \)  
C: \( \frac{1}{2} \)  
D: \( 1 \)  
E: \( 2 \)  
F: \( 5 \)  
G: \( 10 \)  
H: \( 20 \)

**Question 3**  Which of the following statements are true?

i. A scalar ODE of the form \( y^{(M)} + y^{(M-1)} + \cdots + y' = F(y) \), can always be transformed into an equivalent system of ODEs \( u' = f(u) \), with \( u \) a vector of size \( M \).

ii. Discretization errors typically decrease with decreasing step size.

iii. Floating-point rounding errors typically increase with decreasing step size.

iv. If the local discretization error of a scheme is \( O(\Delta t^p) \), its global truncation error is always \( O(\Delta t^{p-1}) \).

A: Unanswered  
B: None  
C: i, ii  
D: i, ii, iii  
E: i, iii, iv  
F: ii, iv  
G: ii, iii, iv  
H: All

**Question 4**  Consider the ODE \( u' = -cu \) with initial condition \( u(0) = 1 \) and constant \( c > 0 \). Using backward Euler:

\[ u_{i+1} = u_i + \Delta t f(u_{i+1}) \]

and \( \Delta t = \frac{1}{2} \), what is the approximation of \( u(4) \)?

A: Unanswered  
B: \( (1 + c/2)^{-8} \)  
C: \( (1 + c/2)^8 \)  
D: \( (1 - c/2)^{-8} \)  
E: \( (1 - c/2)^8 \)  
F: \( (c/2)^{-8} \)  
G: \( (c/2)^8 \)

**Question 5**  Consider the scheme:

\[ u_{i+1} = u_i + \frac{3}{2} \Delta t f(u_i) + \frac{1}{2} \Delta t f(u_{i-1}) \]

where \( u_i \) and \( u_{i-1} \) are known, and \( u_{i+1} \) is unknown. What is the local-truncation error of this scheme? [Hint: Taylor expand \( u_{i-1} \), then \( f(u_{i-1}) \).]
Module 6: Numerical optimization

Question 6  Consider an objective function $J(x)$ for $x \in \mathbb{R}$. Near some unspecified $x_0$, $J$ can be approximated by $J(x_0 + h) = 4 - 2h^2 + O(h^3)$ with $|h| << 1$. What can be said about $J$ at $x_0$? It is a:

A: Unanswered  
B: Local minimum  
C: Local maximum  
D: Global minimum  
E: Global maximum  
F: Zero point (root)  
G: Inflection point  
H: None of the above

Question 7  Consider minimizing $f(x)$. Which of the following is false?

i. The golden-section search is guaranteed to reduce the width interval by a factor of $(1 + \sqrt{5})/2$ at each successful iteration

ii. Newton minimizing $f(x)$ is equivalent to finding a root of $f'(x)$

iii. If Newton converges to a minimum it will converge quadratically

iv. Steepest descent is guaranteed to converge to the global minimum

v. The Nelder-Mead simplex method requires $d+1$ evaluations of $f$ to generate the initial simplex if $x$ is $d$-dimensional

A: Unanswered  
B: i  
C: ii  
D: iii  
E: iv  
F: v  
G: All are true  
H: All are false

Question 8  A rectangular cold storage box with square base of edge length $l$ meters, height $h$ meters and perfectly insulated top has a total volume of $10 \text{ m}^3$. All other sides are uninsulated. The goal is to find $l$ such that heat loss is minimized. Apply Newton’s method for optimization to minimize a suitable objective function (assume heat loss is proportional to surface area). Perform the update in terms of $l$, starting with an initial estimate $l_0 = 1.0$. What is $l_1$ to two decimal places?

A: Unanswered  
B: 0.46  
C: 1.46  
D: 1.19  
E: 1.23  
F: 0.54  
G: 2.46  
H: 1.54

Question 9  Consider $\tilde{x}$ the unique minimum of the function $f(x) = 7x - \ln(x)$. Using Newton’s method for minimization, and starting from $x_0 = 0.1$, what is the error in the approximation after one iteration?

A: Unanswered  
B: 0.129  
C: 0.0129  
D: 0.00129  
E: 0.000129  
F: 0.0000129
**Question 10**  Steepest descent for minimizing $f(x,y)$ in 2-dimensions requires the gradient: $
abla f(x_i, y_i)$ on iteration $i$. Assuming the function $\nabla f(x, y)$ is not known, it can be approximated at $(x_i, y_i)$ by a difference rule (e.g. forward/backward/central differences). What is the minimum number of samples of $f(\cdot)$ required to approximate the gradient at $(x_i, y_i)$ in this way?

A: Unanswered  
B: 1  
C: 2  
D: 3  
E: 4  
F: 5